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Non-Markovian entanglement dynamics between two coupled qubits in the same environment

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Abstract

We analyze the dynamics of the entanglement in two independent non-Markovian channels. In particular, we focus on the entanglement dynamics as a function of the initial states and the channel parameters, such as the temperature and the ratio r between ω_0, the characteristic frequency of the quantum system of interest, and ω_c, the cut-off frequency of the Ohmic reservoir. We give a stationary analysis of the concurrence and find that the dynamic of non-Markovian entanglement concurrence $\mathcal{C}_\rho(t)$ at temperature $k_BT = 0$ is different from the $k_BT > 0$ case. We find that 'entanglement sudden death' (ESD) depends on the initial state when $k_BT = 0$, otherwise the concurrence always disappears at a finite time when $k_BT > 0$, which means that the ESD must happen. The main result of this paper is that the non-Markovian entanglement dynamic is fundamentally different from the Markovian one. In the Markovian channel, entanglement decays exponentially and vanishes only asymptotically, but in the non-Markovian channel the concurrence $\mathcal{C}_\rho(t)$ oscillates, especially in the high temperature case. Then an open-loop controller adjusted by the temperature is proposed to control the entanglement and prolong the ESD time.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement is a remarkable feature of quantum mechanics, and its investigation is both of practical and theoretical significance. It is viewed as a basic resource for quantum information processing (QIP) [1], like realizing high-speed quantum computation [2] and high-security quantum communication [3]. It is also a basic issue in understanding the nature of nonlocality in quantum mechanics [4–6]. However, a quantum system used in quantum information processing inevitably interacts with the surrounding environmental system (or the thermal reservoir), which induces the quantum world into the classical world [7, 14, 21]. Thus, it is an important subject to analyze the entanglement decay induced by the unavoidable interaction with the environment [8–12]. In a one-party quantum system, this process is called decoherence [13–19]. In this paper, we will analyze the entanglement dynamics of a bipartite non-Markovian quantum system. As is well known, the system can only couple to a few environmental degrees of freedom for short times. These will act as memory. In short timescales environmental memory effects always appear in experiments [20]. The characteristic timescales become comparable with the reservoir correlation time in various cases, especially in high-speed communication. Then an exactly analytic description of the open quantum system dynamic is needed, such as quantum Brownian motion (QBM) [21], a two-level atom interacting with a thermal reservoir with Lorentzian spectral density [22], and the devices based on the solid state [23] where memory effects are typically non-negligible. Due to their fundamental importance in quantum information processing and quantum computation, non-Markovian quantum dissipative systems have attracted much attention in recent years [7, 24–28]. Recently, researches on quantum coherence and entanglement influenced and degraded by the external environment have become more and more popular, most of the works contribute to extending the open quantum theory beyond the Markovian approximation [29–31]. In [29], two harmonic oscillators in the quantum domain were studied and their entanglement evolution investigated with the influence of thermal environments. In [30], the dynamics of bipartite Gaussian states in a non-Markovian noisy channel were analyzed. All in all, non-Markovian features of system–reservoir interactions have made great progress, but the theory is far from completion, in particular how the non-Markovian environment influences the system and the difference between Markovian and non-Markovian system evolution are not clear.

In this paper, we will compare the non-Markovian entanglement dynamics with the Markovian one [32] in an Ohmic reservoir with Lorentz–Drude regularization in the following three conditions: $\omega_0 \ll \omega_c$, $\omega_0 \approx \omega_c$ and $\omega_0 \gg \omega_c$, where ω_0 is the characteristic frequency of the quantum system of interest and ω_c is the cut-off frequency of the Ohmic reservoir. Thus, $\omega_c \ll \omega_0$ implies that the spectrum of the reservoir does not completely overlap with the frequency of the system oscillator and $\omega_0 \gg \omega_c$ implies the converse case. Another point of the entanglement dynamics is the temperature. We characterize our system by the low temperature, $k_B T = 0.03\omega_0$, the medium temperature, $k_B T = 3\omega_0$, and the high temperature, $k_B T = 300\omega_0$. We give stationary analysis of the concurrence [9] and find that the dynamics of non-Markovian entanglement concurrence $\mathcal{C}$ at temperature $k_B T = 0$ is fundamentally different from the $k_B T > 0$. We find that 'entanglement sudden death' (ESD) depends on the initial state when $k_B T = 0$, otherwise the concurrence always disappears at finite time when $k_B T > 0$, which means that the ESD must happen. Maniscalco *et al* studied the separability function $S(\tau)$ in [30], where the entanglement oscillation appears for a twin-beam state in non-Markovian channels for high temperature reservoirs. The main result of this paper is that the non-Markovian entanglement dynamics is fundamentally different from the Markovian one. In the Markovian channel, entanglement decays exponentially and vanishes only asymptotically, but in the non-Markovian channel the concurrence $C_\rho(t)$ oscillates, especially in the high temperature case.

The paper is organized as follows. We first introduce the open quantum system and the non-Markovian quantum master equation for driven open quantum systems by the noise and dissipation kernels. In section 3, we introduce Wootters' concurrence and the initial 'X' states. By substituting the initial states into the master equation we get the first-order coupled differential equations, and give the stationary analysis. In section 4, we numerically analyze the Markovian and non-Markovian entanglement dynamics. Then an open-loop controller adjusted by the temperature is proposed to control the entanglement and prolong the ESD time. Conclusions and prospective views are given in section 5.

2. The model

Our system consists of a pair of two-level atoms (two qubits) equally and resonantly coupled to a single cavity mode, with the same coupling strength. The master equation for the reduced density matrix $\rho(t)$ which describes its dynamics is given by [7, 18, 30, 31, 33]

$$\begin{aligned}\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = \frac{\Delta(t)+\gamma(t)}{2}\sum_{j=1}^{2}\left\{2\sigma_j^-\rho\sigma_j^+ - \sigma_j^+\sigma_j^-\rho - \rho\sigma_j^+\sigma_j^-\right\}\\ + \frac{\Delta(t)-\gamma(t)}{2}\sum_{j=1}^{2}\left\{2\sigma_j^+\rho\sigma_j^- - \sigma_j^-\sigma_j^+\rho - \rho\sigma_j^-\sigma_j^+\right\},\end{aligned} \tag{1}$$

where $\sigma^+ = \frac{1}{2}(\sigma_1 + \mathrm{i}\sigma_2)$, $\sigma^- = \frac{1}{2}(\sigma_1 - \mathrm{i}\sigma_2)$, with σ_1, σ_2 being the Pauli matrices. The time-dependent coefficients appearing in the master equation can be written, to the second order in the coupling strength, as follows:

$$\begin{aligned}\Delta(t) &= \int_0^t \mathrm{d}\tau\, k(\tau)\cos(\omega_0\tau),\\ \gamma(t) &= \int_0^t \mathrm{d}\tau\, \mu(\tau)\sin(\omega_0\tau),\end{aligned} \tag{2}$$

with

$$\begin{aligned}k(\tau) &= 2\int_0^\infty \mathrm{d}\omega\, J(\omega)\coth[\omega/2k_BT]\cos(\omega\tau),\\ \mu(\tau) &= 2\int_0^\infty \mathrm{d}\omega\, J(\omega)\sin(\omega\tau)\end{aligned} \tag{3}$$

being the noise and the dissipation kernels, respectively. This master equation (1) is valid for arbitrary temperature. The coefficient $\gamma(t)$ gives rise to a time-dependent damping term, while $\Delta(t)$ gives the diffusive term. The non-Markovian character is contained in the time-dependent coefficients, which contain all the information about the short-time system–reservoir correlations [7]. In the previous equations $J(\omega)$ is the spectral density characterizing the bath,

$$J(\omega) = \frac{\pi}{2}\sum_i \frac{k_i}{m_i\omega_i}\delta(\omega-\omega_i) \tag{4}$$

and the index i labels the different field mode of the reservoir with frequency ω_i. Let the Ohmic spectral density with a Lorentz–Drude cutoff function,

$$J(\omega) = \frac{2}{\pi}\omega\frac{\omega_c^2}{\omega_c^2+\omega^2}, \tag{5}$$

where ω is the frequency of the bath and ω_c is the high-frequency cutoff.

Then the closed analytic expressions for $\Delta(t)$ and $\gamma(t)$ are [18, 31]

$$\gamma(t) = \frac{\omega_0 r^2}{1+r^2}[1 - \mathrm{e}^{-r\omega_0 t}\cos(\omega_0 t) - r\,\mathrm{e}^{-r\omega_0 t}\sin(\omega_0 t)], \tag{6}$$

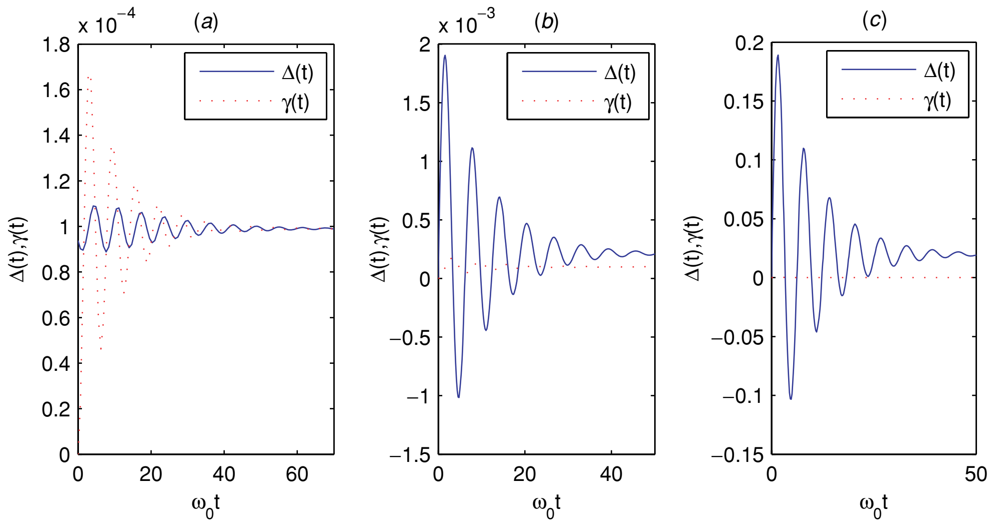

Figure 1. Dynamics of non-Markovian parameters $\Delta(t)$ (blue solid line) and $\gamma(t)$ (red dotted line) at different temperatures: (*a*) $k_B T = 0.01$, (*b*) $k_B T = 1$ and (c) $k_B(t) = 100$, respectively. The other parameters are chosen as $r = 0.1$, $\omega_0 = 1$ and $\alpha^2 = 0.01$.

$$\begin{aligned}\Delta(t) = \omega_0 \frac{r^2}{1+r^2}\Bigg\{&\coth(\pi r_0) - \cot(\pi r_c)\,\mathrm{e}^{-\omega_c t}[r\cos(\omega_0 t) - \sin(\omega_0 t)] \\ &+ \frac{1}{\pi r_0}\cos(\omega_0 t)[\bar{F}(-r_c, t) + \bar{F}(r_c, t) - \bar{F}(\mathrm{i}r_0, t) - \bar{F}(-\mathrm{i}r_0, t)] \\ &- \frac{1}{\pi}\sin(\omega_0 t)\Bigg[\frac{\mathrm{e}^{-\nu_1 t}}{2r_0(1+r_0^2)}[(r_0 - \mathrm{i})\bar{G}(-r_0, t) + (r_0 + \mathrm{i})\bar{G}(r_0, t)] \\ &+ \frac{1}{2r_c}[\bar{F}(-r_c, t) - \bar{F}(r_c, t)]\Bigg]\Bigg\},\end{aligned} \tag{7}$$

where $r_0 = \omega_0/2\pi k_B T$, $r_c = \omega_c/2\pi k_B T$, $r = \omega_c/\omega_0$ and

$$\bar{F}(x, t) \equiv {}_2F_1(x, 1, 1 + x, \mathrm{e}^{-\nu_1 t}), \tag{8}$$

$$\bar{G}(x, t) \equiv {}_2F_1(2, 1 + x, 2 + x, \mathrm{e}^{-\nu_1 t}), \tag{9}$$

$\nu_1 = 2\pi k_B T$, and ${}_2F_1(a, b, c, z)$ is the hypergeometric function. Note that, for time t large enough, the coefficients $\Delta(t)$ and $\gamma(t)$ can be approximated by their Markovian stationary values $\Delta_M = \Delta(t \to \infty)$ and $\gamma_M = \gamma(t \to \infty)$. From equations (6) and (7) we have

$$\gamma_M = \frac{\omega_0 r^2}{1 + r^2} \tag{10}$$

and

$$\Delta_M = \omega_0 \frac{r^2}{1 + r^2} \coth(\pi r_0). \tag{11}$$

Note that $\gamma(t)$ has nothing to do with the temperature [33]. In figure 1, we plot the time evolution of non-Markovian coefficients $\Delta(t)$ and $\gamma(t)$ in different channel temperatures. In figure 1(*a*), the temperature is $k_B T = 0.01$. There are two important main points embodied in the figure, the first is that the coefficient $\gamma(t)$ has dominated the system dissipation at low temperature, the other $\Delta_M = \gamma_M$ in the long time limit. Figures 1(*b*) and (*c*) are the evolution at the medium temperature and high temperature, respectively. The figure shows that the larger the temperature, the more important the coefficient $\Delta(t)$.

3. Concurrence and initial states

In order to describe the entanglement dynamics of the bipartite system, we use the Wootters' concurrence [9, 34]. For a system described by the density matrix ρ, the concurrence $\mathcal{C}(\rho)$ is

$$\mathcal{C}(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \tag{12}$$

where $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are the eigenvalues (with λ_1 being the largest one) of the 'spin-flipped' density operator ζ, which is defined by

$$\zeta = \rho\left(\sigma_y^A \otimes \sigma_y^B\right)\rho^*\left(\sigma_y^A \otimes \sigma_y^B\right), \tag{13}$$

where ρ^* denotes the complex conjugate of ρ and σ_y is the Pauli matrix. $\mathcal{C}$ ranges in magnitude from 0 for a disentanglement state to 1 for a maximally entangled state. The concurrence is related to the entanglement of formation $E_f(\rho)$ by the following relation [34]:

$$E_f(\rho) = \varepsilon[\mathcal{C}(\rho)], \tag{14}$$

where

$$\varepsilon[\mathcal{C}(\rho)] = h\left[\frac{1 + \sqrt{1 - \mathcal{C}^2(\rho)}}{2}\right] \tag{15}$$

and

$$h(x) = -x\log_2 x - (1-x)\log_2(1-x). \tag{16}$$

Assume that the system is initially an 'X' state, which has non-zero elements only along the main diagonal and anti-diagonal. The general structure of an 'X' density matrix is as follows:

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{17}$$

Such states are general enough to include states such as the Werner states, the maximally entangled mixed states (MEMSs) and the Bell states; and it also arises in a wide variety of physical situations [35–37]. This particular form of the density matrix allows us to analytically express the concurrence as [38]

$$\mathcal{C}_{\hat{\rho}}^X = 2\max\{0, K_1, K_2\}, \tag{18}$$

where

$$\begin{aligned} K_1 &= |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}, \\ K_2 &= |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}. \end{aligned} \tag{19}$$

A remarkable aspect of the 'X' states is that the time evolution of the master equation (1) is maintained during the evolution. Substituting (17) into (1), the non-Markovian master equation of the two-qubits system, we obtain the following first-order coupled differential equations:

$$\begin{aligned}
\dot{\rho}_{11}(t) &= -2(\Delta(t)+\gamma(t))\rho_{11}(t) + (\Delta(t)-\gamma(t))\rho_{22}(t) + (\Delta(t)-\gamma(t))\rho_{33}(t),\\
\dot{\rho}_{22}(t) &= (\Delta(t)+\gamma(t))\rho_{11}(t) - 2\Delta(t)\rho_{22}(t) + (\Delta(t)-\gamma(t))\rho_{44}(t),\\
\dot{\rho}_{33}(t) &= (\Delta(t)+\gamma(t))\rho_{11}(t) - 2\Delta(t)\rho_{33}(t) + (\Delta(t)-\gamma(t))\rho_{44}(t),\\
\dot{\rho}_{44}(t) &= (\Delta(t)+\gamma(t))\rho_{22}(t) + (\Delta(t)+\gamma(t))\rho_{33}(t) - 2(\Delta(t)-\gamma(t))\rho_{44}(t),\\
\dot{\rho}_{23}(t) &= -2\Delta(t)\rho_{23}(t),\\
\dot{\rho}_{14}(t) &= -2\Delta(t)\rho_{14}(t).
\end{aligned} \tag{20}$$

From equation (18) the concurrence $\mathcal{C}$ is dependent on the coefficients $\Delta(t\to\infty)$ and $\gamma(t\to\infty)$ in the asymptotic long time limit. Equations (10) and (11) give the stationary value of $\gamma(t)$ and $\Delta(t)$, the Markovian limit

$$\gamma_M \equiv \gamma(t\to\infty) = \frac{\omega_0 r^2}{1+r^2}$$

and

$$\Delta_M \equiv \Delta(t\to\infty) = \omega_0\frac{r^2}{1+r^2}\coth\left(\frac{\omega_0}{2k_BT}\right).$$

γ_M does not depend on temperature, but Δ_M is monotonically increasing with respect to temperature *T*. When $T\to 0$, $\Delta_M \to \frac{\omega_0 r^2}{1+r^2}$. Noting $\coth(\pi r_0) \simeq 1 + \frac{1}{\pi r_0} \simeq \frac{2k_BT}{\omega_0}$, at high temperature

$$\Delta_M^{HT} = 2k_BT\frac{r^2}{1+r^2}. \tag{21}$$

So $\Delta_M > \gamma_M$ is noticeable when temperature $k_BT > 0$. From equations (20) we can get the stationary solution

$$\begin{aligned}
\rho_{11}(t\to\infty) &= \frac{\Delta_M-\gamma_M}{\Delta_M+\gamma_M}\rho_{33}(t\to\infty),\\
\rho_{22}(t\to\infty) &= \rho_{33}(t\to\infty),\\
\rho_{33}(t\to\infty) &= \frac{\Delta_M^2-\gamma_M^2}{4\Delta_M^2},\\
\rho_{44}(t\to\infty) &= \frac{\Delta_M+\gamma_M}{\Delta_M-\gamma_M}\rho_{33}(t\to\infty)
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
\rho_{23}(t\to\infty) &= 0,\\
\rho_{14}(t\to\infty) &= 0.
\end{aligned} \tag{23}$$

According to equations (18) and (19),

$$K_{1,2}(t\to\infty) = 0 - \frac{\Delta_M^2-\gamma_M^2}{4\Delta_M^2} < 0. \tag{24}$$

This means that entanglement must disappear in a finite time period, i.e. the ESD must happen.

When temperature $k_B T = 0, \Delta_M \approx \gamma_M$. From equations (20) we can also get the stationary solution

$$\begin{aligned} \rho_{11}(t \to \infty) &= 0, \\ \rho_{22}(t \to \infty) &= 0, \\ \rho_{33}(t \to \infty) &= 0, \\ \rho_{44}(t \to \infty) &= 1 \end{aligned} \tag{25}$$

and

$$\begin{aligned} \rho_{23}(t \to \infty) &= 0, \\ \rho_{14}(t \to \infty) &= 0. \end{aligned} \tag{26}$$

From equations (18) and (19),

$$K_{1,2}(t \to \infty) = 0. \tag{27}$$

This means that entanglement maybe disappears asymptotically, or oscillates, or other complex behaviors. In the following, we use the numerical methods to demonstrate the concurrence evolution for a special kind of 'X' state, the ρ_{YE} state.

4. Non-Markovian versus Markovian entanglement dynamics

In this section, we use the formalism of the preceding section to determine the disentanglement. As an example, let us consider an important class of mixed states with a single parameter a like the following [27, 39, 40]:

$$\hat{\rho}_{YE} = \frac{1}{3}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1-a \end{pmatrix}. \tag{28}$$

Apparently, the concurrence of ρ_{YE} is $\mathcal{C}_\rho(t) = \max\{0, K(t)\}$ and $K(t) = |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}$. Initially, $\mathcal{C}(\rho(0)) = \frac{2}{3}[1 - \sqrt{a(1-a)}]$. In our simulations, $\omega_0 = 1$ is chosen as the norm unit, and we regard the temperature as a key factor in a disentanglement process, for high temperature $k_B T = 300\omega_0$, intermediate temperature $k_B T = 3\omega_0$ and low temperature $k_B T = 0.03\omega_0$, respectively. Another reservoir parameter playing a key role in the dynamics of the system is the ratio $r = \omega_c/\omega_0$ between the reservoir cutoff frequency ω_c and the system oscillator frequency ω_0. As we will see in this section, by varying these two parameters $k_B T$ and $r = \omega_c/\omega_0$, the time evolution of the open system varies prominently from Markovian to non-Markovian.

In figure 2, the time evolutions of the non-Markovian concurrence for various values of the parameter a in low temperature are plotted. From figure 2 we can see that the entanglement dynamic relies on the different values of $r = \omega_c/\omega_0$. If the spectrum of the reservoir does not completely overlap with the frequency of the system oscillator, $r \ll 1$, we can see from figure 2 that the ESD time is considerable. As the ratio r increases, the ESD time becomes shorter and shorter. With different initial states we can see that the concurrence varies prominently. When the initial state $a = 0$, the non-Markovian entanglement decays slowly, as a increases, the entanglement decays intensely, which means that we can prepare certain initial entanglement states and use this fact to control the system environment in order to prolong the entanglement time.

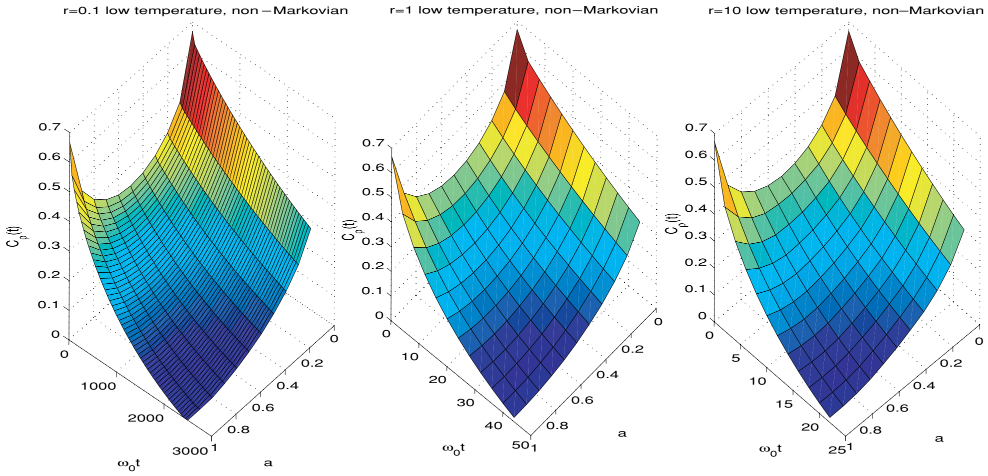

Figure 2. Time evolution of non-Markovian concurrence as a function of parameter 'a' in the low temperature reservoirs.

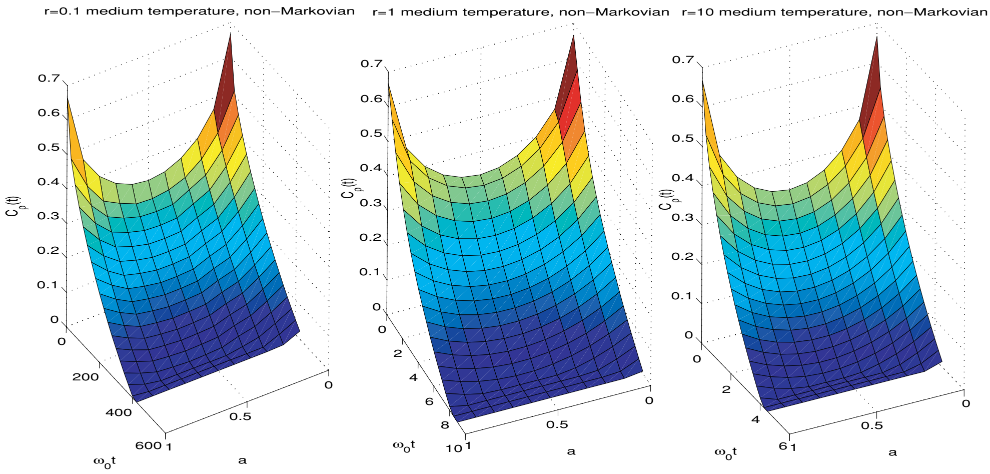

Figure 3. Time evolution of non-Markovian concurrence as a function of parameter 'a' in the medium temperature reservoirs.

Figure 3 is the medium temperature case. Like figure 2, under different systems, different entanglement initial states, corresponding to different values of a, and different r, some decay faster, some slower. But there are some fundamental differences between figures 2 and 3. In section 3, we get the concurrence in the long time limit, and we affirmed that when temperature $k_BT = 0$, the dynamics of non-Markovian entanglement concurrence $\mathcal{C}$ is fundamentally different from the case of $k_BT > 0$. As we can see from figure 3, for 'ρ_{YE}' states, as soon as the temperature larger than zero, the concurrence always disappears at a finite time and there was no long-lived entanglement for any value of a, which means that the ESD must happen. The theoretical proof is $K(t \to \infty) < 0$. But when $k_BT = 0$, the stationary value of $K(t \to \infty)$ equals zero. So, whether or not and when the ESD will happen are not certain at $k_BT = 0$. In figure 4, we give a numerical analysis of entanglement dynamic with

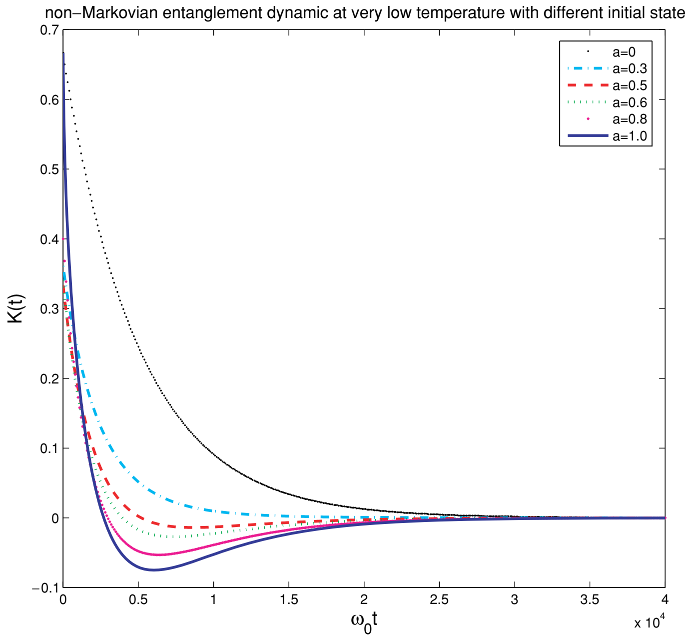

Figure 4. Time evolution of $K(t)$ for temperature $k_B T = 0.000\,001\omega_0$, $r = 0.1$, and initial state $\hat{\rho}_{YE}$ ($a = 0$ (black dotted), $a = 0.3$ (cyan dash-dotted line), $a = 0.5$ (red dash line), $a = 0.6$ (green dotted-dotted line), $a = 0.8$ (magenta asterisk) and $a = 1.0$ (blue solid line)).

different initial states and find that there exists an $\xi \in (0, 1)$, for almost all values of $a > \xi$, the concurrence completely vanishes at a finite time, which is the effect of the ESD. However, for $0 \leqslant a \leqslant \xi$, the entanglement of this state decays exponentially. But when $t \to \infty$, for all initial states, i.e. $a \in [0, 1]$ the concurrence will tend to be 0.

Figure 5 is the high temperature case. One of the remarkable phenomena in this figure is that the ESD time is short. In typical experimental conditions, quantum dots are subjected to an external magnetic field $B \sim 1 - 10T$ [46], the ESD time $t_{\mathrm{ESD}} \sim (3 \times 10^{-1} - 3)/k_B T$. Another obvious phenomenon is in high temperature the Markovian quantum system decays exponentially and vanishes only asymptotically, but in the non-Markovian system the concurrence $C_\rho(t)$ oscillates, which is evidently different from the Markovian. In this case the non-Markovian property becomes evident. This oscillatory phenomenon is induced by the memory effects, which allows the two-qubit entanglement to reappear after a dark period of time. This phenomenon of revival of entanglement after finite periods of 'entanglement death' appears to be linked to the environments single-qubit non-Markovian dynamics, in particular, the $\Delta(t) < 0$ at some times in some environment [31]. The physical conditions examined here are, moreover, more similar to those typically considered in quantum computation, where qubits are taken to be independent and where qubits interact with non-Markovian environments typical of solid state microdevices [41].

As we indicated above, temperature is one of the key factors in the entanglement dynamic. Figures 2–5 are plotted at the chosen temperature, while in figure 6 $k_B T$ ranges from 0 to 100. In figure 6 the concurrence versus 'temperature $k_B T$' versus $\omega_0 t$ in $r = 0.1$, and the initial state is the 'X_{YE}' state with $a = 0$. From figure 6, we can compare the non-Markovian entanglement dynamics with the Markovian one clearly. The left one is the non-Markovian one from which we can see the oscillation of the concurrence. Moreover, at the zero temperature the non-

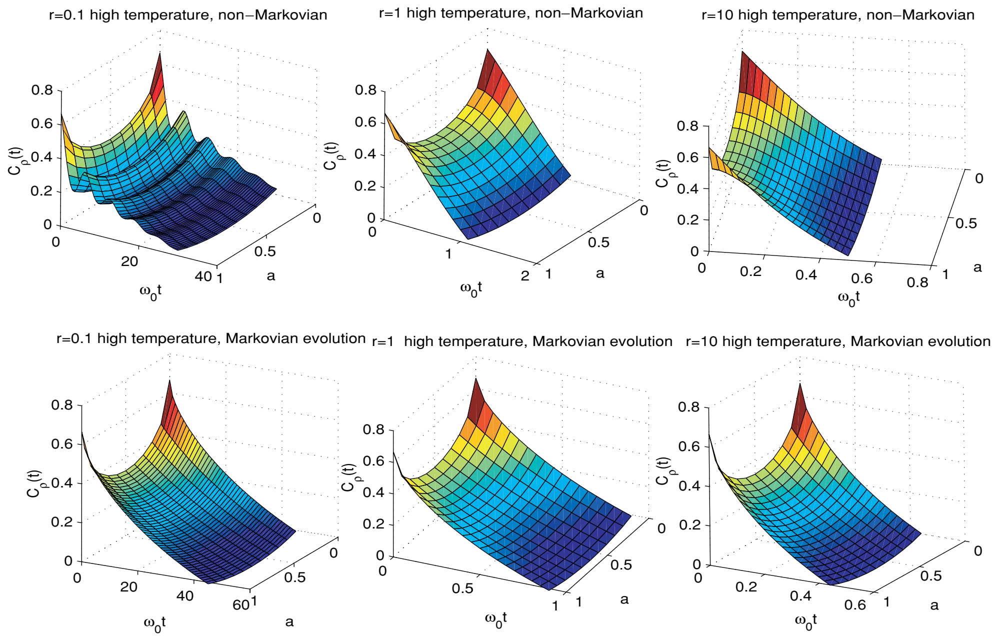

Figure 5. Comparing the non-Markovian entanglement dynamics with the Markovian one by the time evolution of concurrence as a function of parameter 'a' in high temperature reservoirs, at $r = 0.1, r = 1, r = 10$ respectively.

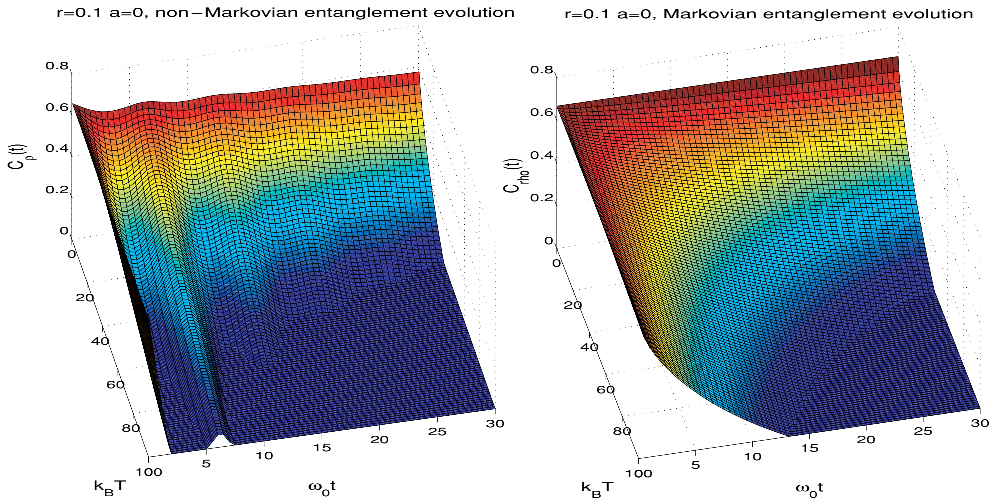

Figure 6. Comparing the non-Markovian entanglement dynamics with the Markovian one by the time evolution of $C_\rho(t)$ as a function of 'k_BT' for initial state $a = 0$ and $r = 0.1$.

Markovian effect is faint, as the temperature rises, the non-Markovian becomes more and more obvious, while the Markovian one decays exponentially. This phenomenon embodies the non-Markovian effect, which is evidently different from the Markovian property. Maniscalco *et al* studied the separability function $S(\tau)$ in [30], where entanglement oscillation appears for the

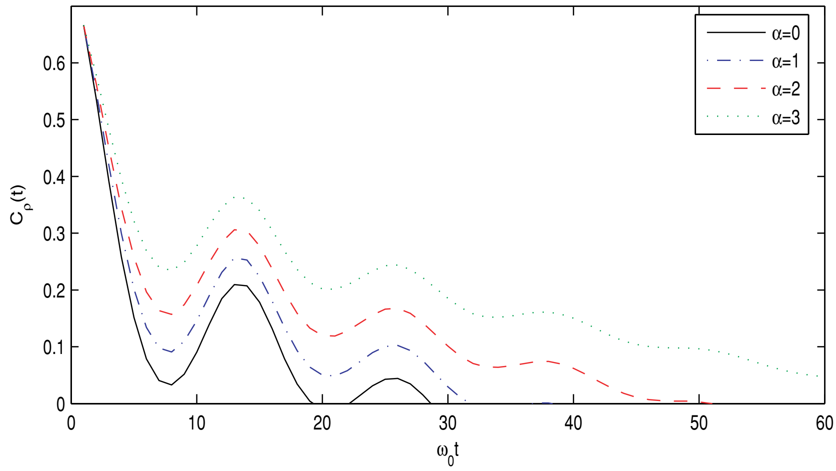

Figure 7. Controlled entanglement evolution with different modulation ($\alpha = 3$ (green dotted line), $\alpha = 2$ (red dashed line), $\alpha = 1$ (blue dashed-dotted line)) and initial evolution (black solid line).

twin-beam state in non-Markovian channels in high temperature reservoirs. Both of them have the same phenomenon. Reference [31] gave a distribution curve when $\Delta(r,t) - \gamma(r,t) > 0$ and $\Delta(r,t) - \gamma(r,t) < 0$. We are convinced that due to the non-Markovian memory effect, particularly $\Delta(t) < 0$ in equations (20), the entanglement concurrence oscillates. With $\Delta(t) - \gamma(t) > 0$ the concurrence descended while $\Delta(t) - \gamma(t) < 0$ the concurrence ascended, which guides us to adjust the temperature to control the entanglement evolution. In order to show this and motivate the related research we design the open loop controller

$$k_B T = \mathrm{e}^{-\alpha|\Delta(t)-\gamma(t)|} k_B T_0, \tag{29}$$

where α is the modulation and $k_B T_0$ is the initial temperature. In figure 7, we plot the controlled entanglement evolution, where the initial temperature is chosen as $k_B T_0 = 30$, which oscillates and ESD occurs at $t \approx 19$. According to figure 1, $\gamma(t)$ can be neglected. For different modulation α, different controlled entanglement evolutions are plotted, and the ESD time can be prolonged for a considerable time.

5. Conclusions

In this paper, we have presented a procedure that allows us to obtain the dynamic of a system consisting of two identical independent qubits, each of them locally interacting with a bosonic reservoir. A non-Markovian master equation between two qubit systems in the same environment was obtained. We characterize our entanglement by the temperature and the ratio r between ω_0, the characteristic frequency of the quantum system of interest, and ω_c, the cut-off frequency of the Ohmic reservoir. For a broad class of initially entangled states, 'X' states, by using Wootters' concurrence, we analyze the long time limit phenomenon of the entanglement dynamic. We find that the dynamic of non-Markovian entanglement concurrence $\mathcal{C}_\rho(t)$ at temperature $k_B T = 0$ is fundamentally different from $k_B T > 0$. When $k_B T = 0$, from our numerical analysis, we find that 'entanglement sudden death' occurs depending on the initial state, but if $k_B T > 0$ the concurrence always disappears at a finite time, which means that ESD must happen. In the $k_B T = 0$ case, we find that there exists an $\xi \in (0, 1)$, for all values of $a > \xi$, the concurrence completely vanishes in a finite time, which is the effect of ESD.

However, for $0 \leqslant a \leqslant \xi$, the entanglement of this state decays exponentially. But when $t \to \infty$, for all initial states, i.e. $a \in [0, 1]$ the concurrence will tend to be 0. From our numerical analysis we also find that the entanglement dynamic relies on the different values of $r = \omega_c/\omega_0$. If $r \ll 1$, the ESD time is considerable. As the ratio r increases, the ESD time becomes shorter and shorter. Moreover, when the initial state $a = 0$, the non-Markovian entanglement decays slowly; as a increases, the entanglement decays intensely. Most of all, we have shown that the non-Markovian dynamics of entanglement, described by concurrence, shows that oscillation even revives after entanglement disappearance, typically for a high temperature non-Markovian system. At last, we design an open loop controller which adjusts the temperature to control the entanglement and prolong the ESD time.

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